

Boltzmann-Uehling-Uhlenbeck calculation of total reaction cross-section and fragmentation cross-section

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Abstract. σ_R , σ_{-n} and σ_{-2n} have been calculated via the BUU model with soft EOS and 0.8 times of $\sigma_{C_{\text{ug}}}$. The density distribution without any adjustable parameters which comes from the RMF model has been introduced into the BUU calculation to replace the normally used one-parameter square-type distribution. The calculated results can reproduce the experimental data well for both halo- and stable-nuclei-induced reactions. Here σ_{-n} or σ_{-2n} is calculated as the difference between σ_R of halo nucleus and core nucleus, by assuming $\sigma_{\text{corr}} \simeq 0$. It indicates that this assumption works very well at high energy in the BUU calculation. More experimental measurements are necessary to test the validity of this assumption at intermediate energy.

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With the development of radioactive ion beams, the properties of nuclei far from the β -stability has been studied intensively. An abnormal enhancement of the interaction cross-section σ_I was observed for ^{11}Li [1]. It indicates that the two loosely bound neutrons are expected to have a very extended density distribution surrounding the ^9Li core, forming the neutron-halo structure. The halo structure of ^{11}Li seems to be consistent with all the experimental results including the enhancement of σ_I [1], the enhancement of the two-neutron removal cross-section σ_{-2n} [2] and the narrow peak in the momentum distribution of the fragmentation ^9Li [3]. Further experimental and theoretical investigations also suggest the existence of neutron or proton haloes in other nuclei [4–20].

It is of particular importance to develop a theoretical method to study the mechanism of exotic-nuclei-induced reactions. A useful tool to study σ_R is the microscopic Glauber multiple-scattering theory [21]. One of the simplest approximation of the Glauber model is the optical-limit approximation. It has been widely used for deducing the nuclear-matter radii from σ_I and σ_R . However, it has been pointed out that it may not be a good approximation if one applies the optical-limit Glauber model to study halo nuclei at intermediate energy [22, 23].

Thus, the few-body limit approximation was introduced into the Glauber model, where a halo nucleus is decomposed into core part and halo part. It found that the few-body limit Glauber model gives a smaller cross-section than that obtained from the optical-limit Glauber model if one uses the same wave function. In the framework of the few-body limit Glauber model, a theory is presented to calculate the nuclear parts of various fragmentation cross-sections, such as σ_R , σ_I , σ_{-n} and σ_{-2n} [23, 24].

On the other hand, the Boltzmann-Uehling-Uhlenbeck (BUU) equation [25, 26] has been introduced into the calculation of σ_R [27]. This model incorporates the Fermi motion, mean field, individual nucleon-nucleon (N - N) interactions and the Pauli-blocking effect simultaneously. It can be used to extract the Equation-Of-State (EOS) and the in-medium N - N cross-section $\sigma_{NN}^{\text{in-medium}}$ via fitting the experimental collective flow and balance energy etc. [28–32]. Within the framework of the BUU model, the average N - N collision number $N(b)$ can be obtained as a function of the impact parameter (b) by assuming a reasonable parameterization of σ_{NN} . According to the Poisson statistics, the nucleon fraction $T_n(b)$ that has experienced n times two-body collisions during the course of nucleus-nucleus reaction can be obtained. The sum of $T_n(b)$ over n ($n \geq 1$) represents the total probability of

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N - N collisions and is related closely to the absorption probability of the nuclear reaction. Therefore, the total reaction cross-section σ_R can be obtained by

$$\sigma_R = 2\pi \int \sum_{n=1}^{\infty} T_n(b) b db = 2\pi \int \left[1 - e^{-N(b)} \right] b db. \quad (1)$$

For simplicity, a square-type density distribution is used to replace the surface-diffused distributions. The width of the square-type density distribution as the unique parameter was obtained by fitting the σ_R 's at relativistic energies. Then the BUU calculations can reproduce the experimental data quite well in a wide energy range. Since the medium effect is different at various incident energies and the real density distribution is more complex than the square-type one, the above assumptions are too simple. It is also interesting to see whether the BUU model can be used to calculate other fragment cross-section, such as σ_{-n} and σ_{-2n} . In this letter, we apply the BUU model to investigate σ_R , σ_{-n} and σ_{-2n} simultaneously, by using the density distributions which are calculated from the nonlinear relativistic mean-field model (RMF) [33].

Let us consider the reaction of ^{11}Li with a target nucleus T , as a typical example. The reaction can be summarized in the frame as [23, 24]

$$\begin{aligned} &^{11}\text{Li}\{|\Psi_0\rangle\} + T\{|\mathbf{K}, \Theta_0\rangle\} \rightarrow \\ &^{11}\text{Li}\{|\mathbf{q}, \Psi_\alpha\rangle\} + T\{|\mathbf{K} - \mathbf{q}, \Theta_\beta\rangle\}, \end{aligned} \quad (2)$$

where the initial ^{11}Li with the intrinsic wave function Ψ_0 is at rest in the projectile rest frame; Θ_0 is the intrinsic wave function of the target nucleus and $-\hbar\mathbf{K}$ is the relative momentum. At the final stage of the reaction, ^{11}Li goes to the state Ψ_α specified by α with momentum transfer $-\hbar\mathbf{q}$. The target nucleus receives a momentum transfer $-\hbar\mathbf{q}$ and goes to the state β . It is defined that $\alpha = 0$ and $\beta = 0$ stand for the respective ground states. σ_R is obtained by summing the cross-section of the reaction over all possible final states ($\alpha\beta$), except for $\alpha\beta = 00$. σ_I is obtained by summing all possible states except for $\alpha = 0$. More details can be found in refs. [23, 24].

By assuming the core-plus-halo neutrons structure of ^{11}Li , a basic relation is established between σ_I or σ_R and σ_{-2n} [23]; we have

$$\begin{aligned} \sigma_{-2n}(^{11}\text{Li} + T) &= \sigma_I(^{11}\text{Li} + T) - \sigma_I(^9\text{Li} + T) \\ &= \sigma_R(^{11}\text{Li} + T) - \sigma_R(^9\text{Li} + T) \\ &\quad + \sigma_{\text{corr}}(^{11}\text{Li} \rightarrow ^9\text{Li}) \end{aligned} \quad (3)$$

with the correction term

$$\begin{aligned} \sigma_{\text{corr}}(^{11}\text{Li} \rightarrow ^9\text{Li}) &= \sigma_{\text{diff}}(^9\text{Li} + T) - \sigma_{\text{diff}}(^{11}\text{Li} + T) \\ &= (\sigma_R(^9\text{Li} + T) - \sigma_I(^9\text{Li} + T)) \\ &\quad - (\sigma_R(^{11}\text{Li} + T) - \sigma_I(^{11}\text{Li} + T)). \end{aligned} \quad (4)$$

Ogawa *et al.* [23] estimated the difference between σ_R and σ_I (σ_{diff}) and found that it is less than a few percent at high energy. It strongly suggests that the contribution of $\sigma_{\text{corr}}(^{11}\text{Li} \rightarrow ^9\text{Li})$ to $\sigma_{-2n}(^{11}\text{Li} + T)$ is very small. Then it

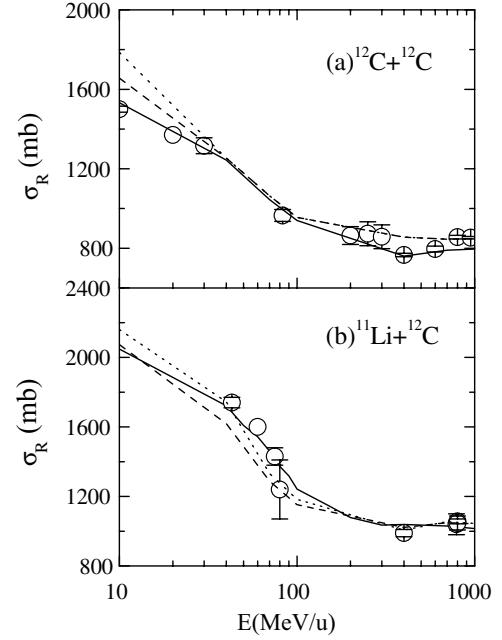


Fig. 1. The variation of σ_R with incidence energy for $^{12}\text{C} + ^{12}\text{C}$ and $^{11}\text{Li} + ^{12}\text{C}$ reaction systems, respectively. Open circles represent the experimental data taken from literatures, while the solid curves represent our BUU calculation.

will be a good approximation to assume that σ_{corr} is zero at high energy. However, σ_{diff} is expected to be larger at an energy lower than 100 MeV/u. The contribution of $\sigma_{\text{corr}}(^{11}\text{Li} \rightarrow ^9\text{Li})$ to $\sigma_{-2n}(^{11}\text{Li} + T)$ should be considered carefully in the intermediate-energy range. In this calculation, σ_{-2n} is approximate as the difference between the σ_R 's of ^{11}Li and ^9Li as given by

$$\sigma_{-2n}(^{11}\text{Li} + T) \simeq \sigma_R(^{11}\text{Li} + T) - \sigma_R(^9\text{Li} + T). \quad (5)$$

Here soft EOS and 0.8 times of Cugnon parameterization of the nucleon-nucleon cross-section (σ_{Cug}) [27] are used in the BUU framework. The density distribution, which is the most important input of the BUU model, is calculated by a nonlinear relativistic mean-field model (RMF) [34–38] with the NLZ force parameter [38]. More detailed studies indicate that RMF calculations with other sets of force parameters [33, 39, 40] will give similar density distributions. It is also noticed that there is no adjustable parameter in the present BUU calculation.

As shown in fig. 1, the open circles are the experimental σ_R of $^{12}\text{C} + ^{12}\text{C}$ and $^{11}\text{Li} + ^{12}\text{C}$ reaction systems, respectively. The solid lines indicate the present BUU calculation by using the RMF-calculated density distribution. The dashed and dotted lines indicate the BUU calculation by using square-type and HO-type density distributions, respectively, where the width of each density distribution as the unique parameter is obtained by fitting σ_R at relativistic energy. The density distributions of ^{11}Li and ^{12}C which are given by the RMF theory with the NLZ interaction are plotted in fig. 2. The solid, dashed and dotted curves are the density distribution of matter, proton and neutron, respectively. It seems that the RMF theory

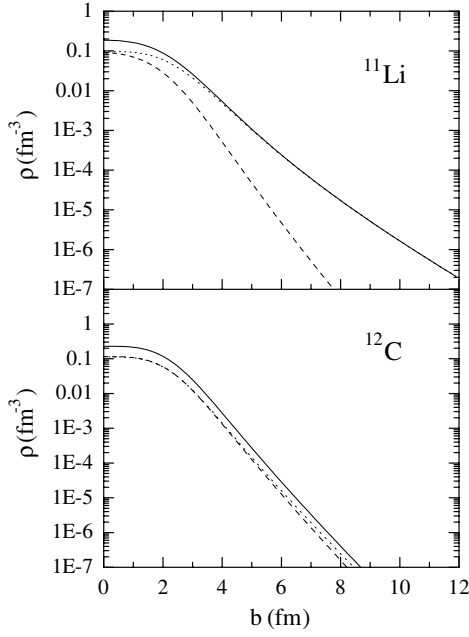


Fig. 2. Density distributions of matter, neutron and proton of ^{11}Li and ^{12}C in the RMF theory with the NLZ interaction. The solid, dashed and dotted curves are the density distributions of matter, proton and neutron, respectively.

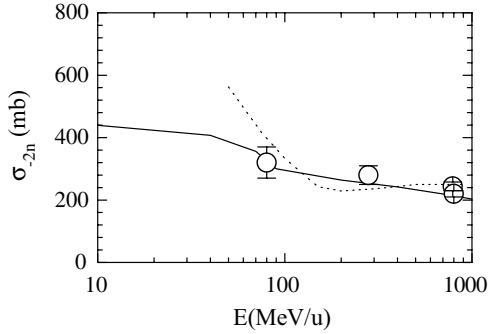


Fig. 3. Energy dependence of σ_{-2n} of $^{11}\text{Li} + ^{12}\text{C}$. The open circles are experimental data. The solid curve is the present BUU calculation. The dotted curve is the Glauber-calculated result of ref. [41].

can describe the long tail of the neutron density distribution of ^{11}Li [33,39]. It can be seen from fig. 1 that the present BUU calculations reproduce the experimental σ_{R} very well, by introducing the more natural density distribution without any adjustable parameters to replace the normally used square-type or HO-type distributions with one adjustable width parameter.

$\sigma_{-2n}(^{11}\text{Li} + ^{12}\text{C})$ is calculated by using eq. (5) in the framework of the BUU model, where σ_{corr} is assumed to be zero. It should be noted that the calculated values only correspond to the nuclear part of the measured σ_{-2n} values. Figure 3 shows the energy dependence of σ_{-2n} of the $^{11}\text{Li} + ^{12}\text{C}$ reaction system. The experimental data are indicated by open circles [2,23]. The solid curve is the present calculation. The dashed curve is the calculated result which is taken from ref. [41]. It can be seen that the

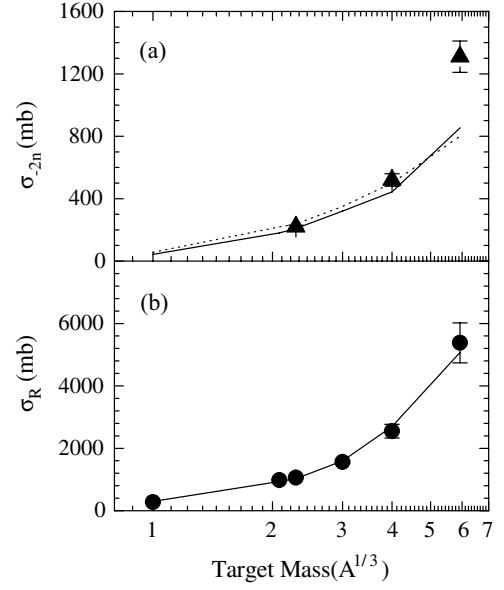


Fig. 4. (a) The variation of σ_{-2n} of ^{11}Li at 800 MeV/u with $A^{1/3}$. (b) The variation of σ_{R} of ^{11}Li at 800 MeV/u with $A^{1/3}$. All solid symbols are experimental data. The solid curves are the present BUU calculation. The dotted curve is the calculated result of ref. [24].

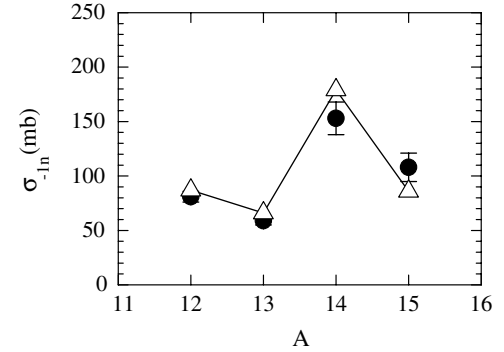


Fig. 5. Mass number dependence of σ_{-n} induced by B isotopes. The solid circles indicate the experimental data. The solid curve connecting open triangles indicates the present BUU calculation.

BUU-calculated σ_{-2n} decreases gently with increasing energy both in the low- and high-energy range. It suggests that the assumption ($\sigma_{\text{corr}} \simeq 0$) works well at high energy. At intermediate energy, the calculated σ_{-2n} of ref. [41] increases fast with the decrease of energy, while the BUU calculation gives a slight increasing trend. However, the experimental data of σ_{-2n} in this energy range is rather rare. More experimental measurements are necessary to estimate the roles of different mechanisms simultaneously and test the validity of the assumption ($\sigma_{\text{corr}} \simeq 0$) at intermediate energy.

The variations of σ_{-2n} and σ_{R} of ^{11}Li at 800 MeV/u with various targets are plotted as a function of $A^{1/3}$, as shown in fig. 4(a) and (b), respectively. All solid symbols indicate the experimental data. The solid curves are the present results. The dotted curve is the calculated result

Table 1. Coulomb dissociation cross-section of $^{11}\text{Li} + ^{64}\text{Cu}$ and $^{11}\text{Li} + ^{208}\text{Pb}$ at 800 MeV/ u .

Reaction system	$^{11}\text{Li} + \text{Cu}$	$^{11}\text{Li} + \text{Pb}$
[41] (mb)	81	765
[44] (mb)	–	971
[45] (mb)	–	960
[43] (mb)	89 ± 3	635 ± 25
This work (mb)	76 ± 40	457 ± 100

of the Glauber model [24]. The present calculation gives the similar trend of σ_{-2n} as the result of the Glauber model [24]. The nice agreement between theory and experiment is obtained for light reaction system. It also confirms that the assumption ($\sigma_{\text{corr}} \simeq 0$) works at high energy. It is noticed that the present BUU calculation only includes the nuclear-part contribution, whereas the experimental values include both the nuclear and Coulomb parts contributions. For the $^{11}\text{Li} + ^{208}\text{Pb}$ reaction system, the BUU-calculated nuclear part of σ_{-2n} is 853 mb, which is close to the value calculated in the diffractive eikonal model [42]. The Coulomb dissociation cross-section can be deduced as the difference of experimental σ_{-2n} and the BUU-calculated nuclear part of σ_{-2n} . It is found that the deduced Coulomb contribution of σ_{-2n} can be neglected for a light target. It is inferred from the recent study [23] that the nuclear-Coulomb interference would become more important at $b \simeq 9 \sim 20$ fm. Thus, for the $^{11}\text{Li} + ^{208}\text{Pb}$ reaction system, the Coulomb effect on σ_{-2n} plays a more important role than for other light reaction systems. As shown in table 1, the deduced Coulomb dissociation cross-section of $^{11}\text{Li} + ^{64}\text{Cu}$ is 76 ± 40 mb in the BUU calculation, which is equal to other calculations [41,43]. For $^{11}\text{Li} + ^{208}\text{Pb}$, the present result is smaller than other estimations [41,44,45,43]. More studies about both Coulomb and nuclear contributions will be necessary in the future.

One-neutron removal cross-sections σ_{-n} of B isotopes [46] at 43–76 MeV/ u are further studied by the above BUU model, as plotted in fig. 5. Here the RMF-calculated density distributions of B isotopes are used. It can be seen that the BUU calculation can reproduce the isospin dependence of experimental data. The σ_{-n} of ^{14}B is larger than its neighbors, which may be attributed to the weak binding of the last neutron ($S_n = 0.97$ MeV) and the extended valence density distribution of ^{14}B . Together with the enhancement of the σ_R measurement for ^{14}B [7,8] and the ground-state structure [47,48], it supports strongly the conclusion that there exists a one-neutron halo structure in ^{14}B . It suggests that the BUU calculation can be used to study the isospin dependence of σ_R , σ_{-n} or σ_{-2n} simultaneously and give a useful criterion of the halo structure.

In conclusion, σ_R , σ_{-n} and σ_{-2n} have been calculated via the BUU model by using soft EOS and 0.8 times of σ_{Cug} . The density distributions which come from the RMF model with the NLZ interaction have been introduced into the BUU model to replace the normally used square-type

or HO-type density distributions. Then the BUU calculations with no adjustable parameter can reproduce well the experimental σ_R , σ_{-n} and σ_{-2n} for various reaction systems. Here σ_{-n} and σ_{-2n} have been calculated as the difference between σ_R of the halo nucleus and core nucleus, by assuming $\sigma_{\text{corr}} \simeq 0$. It indicates that this assumption works well at high energy, which was also mentioned by refs. [23,49]. At intermediate energy, it seems that this assumption still works and more experimental data is necessary to obtain a possible conclusion. It suggests that the BUU model can also be used to study the momentum distribution of fragmentation, by combining with the judgement of fragments. Thus, it is interesting to use the BUU model to test the validity and consistency of the enhancement of σ_R , the enhancement of σ_{-n} and the narrow momentum distribution of fragmentation as criterions of the halo structure simultaneously. This work is in process.

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